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ON RADIAL SYMMETRIC n-SIGRAPHS

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Abstract

An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ ($\mu : V \to H_n$) is a function. In this paper, we introduced a new notion radial symmetric n-sigraph of a symmetric n-sigraph and its properties are obtained. Also, we obtained the structural characterization of radial symmetric n-signed graphs.

1. Introduction

Unless mentioned or defined otherwise, for all terminology and notion in graph theory the reader is refer to [1]. We consider only finite, simple graphs free from self-loops.

Key Words and Phrases : *Symmetric n-sigraphs, Symmetric n-marked graphs, Balance, Switching, Radial symmetric n-sigraphs, Complementation.*
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Let $n \geq 1$ be an integer. An *n-tuple* $(a_1, a_2, ..., a_n)$ is *symmetric*, if $a_k = a_{n-k+1}, 1 \leq k \leq n$. Let $H_n = \{(a_1, a_2, ..., a_n) : a_k \in \{+, -\}, a_k = a_{n-k+1}, 1 \leq k \leq n\}$ be the set of all symmetric n-tuples. Note that H_n is a group under coordinate wise multiplication, and the order of H_n is 2^m, where $m = \lceil \frac{n}{2} \rceil$.

A *symmetric n-sigraph (symmetric n-marked graph)* is an ordered pair $S_n = (G, \sigma)$ ($S_n = (G, \mu)$), where $G = (V, E)$ is a graph called the *underlying graph* of S_n and $\sigma : E \to H_n$ ($\mu : V \to H_n$) is a function.

In this paper by an *n-tuple/n-sigraph/n-marked graph* we always mean a symmetric n-tuple/symmetric n-sigraph/symmetric n-marked graph.

An n-tuple $(a_1, a_2, ..., a_n)$ is the *identity n-tuple*, if $a_k = +$, for $1 \leq k \leq n$, otherwise it is a *non-identity n-tuple*. In an n-sigraph $S_n = (G, \sigma)$ an edge labelled with the identity n-tuple is called an *identity edge*, otherwise it is a *non-identity edge*.

Further, in an n-sigraph $S_n = (G, \sigma)$, for any $A \subseteq E(G)$ the *n-tuple* $\sigma(A)$ is the product of the n-tuples on the edges of A.

In [10], the authors defined two notions of balance in n-sigraph $S_n = (G, \sigma)$ as follows (See also R. Rangarajan and P.S.K.Reddy [6]):

Definition. Let $S_n = (G, \sigma)$ be an n-sigraph. Then,

(i) S_n is *identity balanced* (or *i-balanced*), if product of n-tuples on each cycle of S_n is the identity n-tuple, and

(ii) S_n is *balanced*, if every cycle in S_n contains an even number of non-identity edges.

Note: An i-balanced n-sigraph need not be balanced and conversely.

The following characterization of i-balanced n-sigraphs is obtained in [10].

Proposition 1*(E. Sampathkumar et al. [10])* : An n-sigraph $S_n = (G, \sigma)$ is i-balanced if, and only if, it is possible to assign n-tuples to its vertices such that the n-tuple of each edge uv is equal to the product of the n-tuples of u and v.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S_n defined as follows: each vertex $v \in V$, $\mu(v)$ is the n-tuple which is the product of the n-tuples on the edges incident with v. *Complement* of S_n is an n-sigraph $\overline{S_n} = (\overline{G}, \sigma^c)$, where for any edge $e = uv \in \overline{G}$, $\sigma^c(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ as defined here is an i-balanced n-sigraph due to Proposition 1 [12].

In [10], the authors also have defined switching and cycle isomorphism of an n-sigraph $S_n = (G, \sigma)$ as follows: (See also [4,7-9, 12-22]

Let $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$, be two n-sigraphs. Then S_n and S'_n are said to be *isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that if uv is an edge in S_n with label $(a_1, a_2, ..., a_n)$ then $\phi(u)\phi(v)$ is an edge in S'_n with label $(a_1, a_2, ..., a_n)$.

Given an n-marking μ of an n-sigraph $S_n = (G, \sigma)$, *switching* S_n with respect to μ is the operation of changing the n-tuple of every edge uv of S_n by $\mu(u)\sigma(uv)\mu(v)$. The n-sigraph obtained in this way is denoted by $\mathcal{S}_\mu(S_n)$ and is called the *μ-switched n-sigraph* or just *switched n-sigraph*.

Further, an n-sigraph S_n *switches* to n-sigraph S'_n (or that they are *switching equivalent* to each other), written as $S_n \sim S'_n$, whenever there exists an n-marking of S_n such that $\mathcal{S}_\mu(S_n) \cong S'_n$.

Two n-sigraphs $S_n = (G, \sigma)$ and $S'_n = (G', \sigma')$ are said to be *cycle isomorphic*, if there exists an isomorphism $\phi : G \to G'$ such that the n-tuple $\sigma(C)$ of every cycle C in S_n equals to the n-tuple $\sigma(\phi(C))$ in S'_n.

We make use of the following known result (see [10]).

Proposition 2 ***(E. Sampathkumar et al. [10])*** : Given a graph G, any two n-sigraphs with G as underlying graph are switching equivalent if, and only if, they are cycle isomorphic.

Let $S_n = (G, \sigma)$ be an n-sigraph. Consider the n-marking μ on vertices of S defined as follows: each vertex $v \in V$, $\mu(v)$ is the product of the n-tuples on the edges incident at v. *Complement* of S is an n-sigraph $\overline{S_n} = (\overline{G}, \sigma')$, where for any edge $e = uv \in \overline{G}$, $\sigma'(uv) = \mu(u)\mu(v)$. Clearly, $\overline{S_n}$ as defined here is an i-balanced n-sigraph due to Proposition 1.

2. Radial n-Sigraph of an n-Sigraph

Kathiresan and Marimuthu [2] introduced a new type of graph called radial graph. Two vertices of a graph Γ are said to be radial to each other if the distance between them is equal to the radius of the graph. The radial graph of a graph G, denoted by $R(G)$, has the vertex set as in G and two vertices are adjacent in $R(G)$ if, and only if, they are radial in G. If G is disconnected, then two vertices are adjacent in $R(G)$ if they belong to different components of G. A graph G is called a *radial graph* if $R(G') = \Gamma$ for some graph G'.

Motivated by the existing definition of complement of an n-sigraph, we extend the notion of radial graphs to n-sigraphs as follows:

The *radial n-sigraph* $R(S_n)$ of an n-sigraph $S_n = (G, \sigma)$ is an n-sigraph whose underlying graph is $R(G)$ and the n-tuple of any edge uv is $R(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n. Further, an n-sigraph $S_n = (G, \sigma)$ is called radial n-sigraph, if $S_n \cong R(S'_n)$ for some n-sigraph S'_n. The following result indicates the limitations of the notion $R(S_n)$ as introduced above, since the entire class of i-unbalanced n-sigraphs is forbidden to be radial n-sigraphs.

Propositiion 3 : For any n-sigraph $S_n = (G, \sigma)$, its radial n-sigraph $R(S_n)$ is i-balanced.

Proof : Since the n-tuple of any edge uv in $R(S_n)$ is $\mu(u)\mu(v)$, where μ is the canonical n-marking of S_n, by Proposition 1, $R(S_n)$ is i-balanced.

For any positive integer k, the k^{th} iterated radial n-sigraph $R(S_n)$ of S_n is defined as follows:

$$(R)^0(S_n) = S_n, (R)k(S_n) = R((R)^{k-1}(S_n))$$

Corollary 4 : For any n-sigraph $S_n = (G, \sigma)$ and any positive integer k, $(R)^k(S_n)$ is i-balanced.

The following result characterize n-sigraphs which are radial n-sigraphs.

Theorem 5 : An n-sigraph $S_n = (G, \sigma)$ is a radial n-sigraph if, and only if, S_n is i-balanced n-sigraph and its underlying graph G is a radial graph.

Proof : Suppose that S_n is i-balanced and G is a $R(G)$. Then there exists a graph H such that $R(H) \cong G$. Since S_n is i-balanced, by Proposition 1, there exists an n-marking μ of G such that each edge uv in S_n satisfies $\sigma(uv) = \mu(u)\mu(v)$. Now consider the n-sigraph $S'_n = (H, \sigma')$, where for any edge e in H, $\sigma'(e)$ is the n-marking of the corresponding vertex in G. Then clearly, $R(S'_n) \cong S_n$. Hence S_n is a radial n-sigraph.

Conversely, suppose that $S_n = (G, \sigma)$ is a radial n-sigraph. Then there exists an n-sigraph $S'_n = (H, \sigma')$ such that $R(S'_n) \cong S_n$. Hence G is the $R(G)$ of H and by Proposition 3, S_n is i-balanced. □

The following result characterizes the n-sigraphs which are isomorphic to radial n-sigraphs. In case of graphs the following result is due to Kathiresan and Marimuthu [3] :

Theorem 6 : Let $G = (V, E)$ be a graph of order n. Then $R(G) \cong G$ if, and only if, G is a connected graph with $r(G) = d(G) = 1$ or $r(G) = 1$ and $d(G) = 2$.

Theorem 7 : For any n-sigraph $S_n = (G, \sigma)$, the radial n-sigraph $R(S_n)$ and S_n are switching equivalent if, and only if, S_n is i-balanced n-sigraph and the underlying G with $r(G) = d(G) = 1$ or $r(G) = 1$ and $d(G) = 2$.

Proof : Suppose $S_n \sim R(S_n)$. This implies, $G \cong R(G)$ and hence G is a graph with $r(G) = d(G) = 1$ or $r(G) = 1$ and $d(G) = 2$, Proposition 3 implies that $R(S_n)$ is i-balanced and hence if S_n is i-unbalanced and its $R(S_n)$ being i-balanced can not be switching equivalent to S_n in accordance with Proposition 2. Therefore, S_n must be i-balanced.

Conversely, suppose that S_n is an i-balanced n-sigraph and its undrelying G with $r(G) = d(G) = 1$ or $r(G) = 1$ and $d(G) = 2$. Then, since $R(S_n)$ is i-balanced as per Proposition 3 and since $G \cong R(G)$, the result follows from Proposition 2 again. □

In [3], the authors characterize the graphs for which $R(G) \cong \overline{G}$.

Theorem 8 : Let G be a graph of order n. Then $R(G) \cong \overline{G}$ if, and only if, either $S_2(G) = V(G)$ or G is disconnected in which each component is complete.

In view of the above result, we have the following result that characterizes the family of n-sigraphs satisfies $R(S_n) \sim \overline{S_n}$.

Theorem 9 : For any n-sigraph $S_n = (G, \sigma)$, $R(S_n) \sim \overline{S_n}$ if, and only if, either $S_2(G) = V(G)$ or G is disconnected in which each component is complete.

Proof : Suppose that $R(S_n) \sim \overline{S_n}$. Then clearly, $R(G) \cong \overline{G}$. Hence by Theorem 8, G is either $S_2(G) = V(G)$ or disconnected in which each component is complete.

Conversely, suppose that S_n is an n-sigraph whose underlying graph is either $S_2(G) = V(G)$ or G is disconnected in which each component is complete. Then by Theorem 8, $R(G) \cong \overline{G}$. Since for any n-sigraph S_n, both $R(S_n)$ and $\overline{S_n}$ are i-balanced, the result follows by Proposition 2. □

The following result due to Kathiresan and Marimuthu [3] gives a characterization of graphs for which $R(G) \sim R(\overline{G})$.

Theorem 10 : Let G be a graph. Then $R(G) \sim R(\overline{G})$ if, and only if, G satisfies any one the following conditions:

i. G or $\overline{G}$ is complete.

ii. G or $\overline{G}$ is disconnected with each component complete out of which one is an isolated vertex.

We now give a characterization of n-sigraphs whose radial n-sigraphs are switching equivalent to their radial n-sigraph of complementary n-sigraphs.

Theorem 11 : For any n-sigraph $S_n = (G, \sigma)$, $R(S_n) \sim R(\overline{S_n})$ if, and only if, G satisfies the conditions of Theorem 10.

Theorem 12 : For any two S_n and S'_n with the same underlying graph, their radial n-sigraphs are switching equivalent.

For any $m \in H_n$, the *m-complement* of $a = (a_1, a_2, ..., a_n)$ is: $a^m = am$. For any $M \subseteq H_n$, and $m \in H_n$, the *m-complement* of M is $M^m = \{a^m : a \in M\}$.

For any $m \in H_n$, the *m-complement* of an n-sigraph $S_n = (G, \sigma)$, written (S_n^m), is the same graph but with each edge label $a = (a_1, a_2, ..., a_n)$ replaced by a^m.

For an n-sigraph $S_n = (G, \sigma)$, the $R(S_n)$ is i-balanced. We now examine, the condition under which m-complement of $R(S_n)$ is i-balanced, where for any $m \in H_n$.

Proposition 13 : Let $S_n = (G, \sigma)$ be an n-sigraph. Then, for any $m \in H_n$, if $R(G)$ is bipartite then $(R(S_n))^m$ is i-balanced.

Proof: Since, by Proposition 3, $R(S_n)$ is i-balanced, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $R(S_n)$ whose k^{th} co-ordinate are $-$ is even. Also, since $R(G)$ is bipartite, all cycles have even length; thus, for each k, $1 \leq k \leq n$, the number of n-tuples on any cycle C in $R(S_n)$ whose k^{th} co-ordinate are $+$ is also even. This implies that the same thing is true in any m-complement, where for any $m, \in H_n$. Hence $(R(S_n))^t$ is i-balanced. □

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